

## Proton Momentum Distributions and Elastic Electron Scattering form Factors for Some Even-A $1f-2p$ Shell Nuclei

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### Abstract

The proton momentum distributions (PMD) and elastic charge form factors,  $F(q)$ , of the ground state for some even mass nuclei in the  $1f-2p$  shell, such as  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei have been calculated in the framework of the Coherent Density Fluctuation Model (CDFM) and expressed in terms of the fluctuation function (weight function) ( $|f(x)|^2$ ). The fluctuation function has been related to the charge density distribution (CDD) of the nuclei and obtained from the theory and experiment. The feature of the long-tail behavior at high momentum region of the proton momentum distributions has been determined by both the theoretical and experimental fluctuation functions. The observed electron scattering form factors for  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei are in very good agreement with the present calculations throughout all values of momentum transfer  $q$ .

Keywords: Charge density distributions, Proton momentum distributions, Elastic electron scattering form factor, Root mean square radii.

### Introduction

Nuclear size and density distribution are the basic quantities that describe the nuclear properties [1-3]. The charge densities can give us much detailed information on the internal structure of nuclei since they are directly related to the proton wave functions, which are important keys for many calculations in nuclear physics. Electron-nucleus scattering is known to be one of the powerful tools for investigating nuclear charge density distributions. Charge density distributions for stable nuclei have been well studied with this method [4-6]. The electron-nucleus interaction is considered [7] by the first Born approximation as an exchange of a virtual photon. In this case the initial and final particles are considered free and can be represented by plane waves. The first Born approximation is being valid only if  $Z\alpha \ll 1$ , where  $Z$  is the atomic number and  $\alpha$  is the fine structure constant. According to this approximation the interaction of the electron with the charge distribution of the nucleus is considered as an exchange of a virtual photon with zero angular momentum along the direction of the momentum transfer  $q$ ; this is called "Coulomb or longitudinal scattering". In addition to the electron scattering experiments, the scattering of ions and particles from nuclei has provided along the years invaluable

information on charge, matter and current on stable nuclei and near the stability line. Also, experiments on hadron elastic scattering and total cross section measurements provided information about the nuclear matter density distribution. Additionally, from the momentum distribution of the fragments from break-up reactions the rms nuclear matter radii can be determined [8].

In coherent density fluctuations model (CDFM), which is exemplified by the work of Antonov *et al.* [9,10,11], the local nucleon density distribution (NDD) and the nucleon momentum distributions (NMD) are simply related and expressed in terms of experimentally obtainable fluctuation function (weight function)  $|f(x)|^2$ . They [9,10,11] studied the NMD of ( $^4\text{He}$  and  $^{16}\text{O}$ ),  $^{12}\text{C}$  and ( $^{39}\text{K}$ ,  $^{40}\text{Ca}$  and  $^{48}\text{Ca}$ ) nuclei using weight functions  $|f(x)|^2$  specified by the two parameter Fermi (2PF) NDD [12], the data of Reuter *et al.* [13] and the model independent NDD [12], respectively. It is significant to remark that all above studies, employed the framework of the CDFM, proved a high momentum tail in the NMD. Elastic electron scattering from  $^{40}\text{Ca}$  nucleus was also investigated in Ref. [9], where the calculated elastic differential cross sections ( $d\sigma/d\Omega$ ) are in good agreement with those of experimental data.

Nearly all the CDFM investigations are based on the use of weight functions expressed in terms of the experimental NDD. In the present study, we utilize the CDFM with weight functions expressed in terms of theoretical CDD. We first try to derive a theoretical form for the CDD, applicable throughout all *fp*-shell nuclei, based on the use of the single particle harmonic oscillator wave function and the occupation numbers of the states. The derived form of the CDD is employed in determining the theoretical weight function  $|f(x)|^2$  which is then used in the CDFM to study the proton momentum distribution (PMD) for some *fp*-shell nuclei with even A, such as,  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei. It is found that the theoretical weight function  $|f(x)|^2$  based on the derived CDD is capable to give information about the PMD and elastic charge form factors as do those of the experimental data.

**Theory**

The charge density distribution (CDD) of one body operator can be written as [14,15]:

$$\rho_{ch}(r) = \frac{1}{4\pi} \sum_{n\ell} \xi_{n\ell} 2(2\ell + 1) |R_{n\ell}|^2 \dots\dots (1)$$

where  $\xi_{n\ell}$  is the proton occupation probability of the state  $n\ell$  ( $\xi_{n\ell} = 0$  or 1 for closed shell nuclei and  $0 < \xi_{n\ell} < 1$  for open shell nuclei) and  $R_{n\ell}$  is the radial part of the single particle harmonic oscillator wave function. In the simple shell model, the  $1f - 2p$  shell nuclei are considered as an inert core of filled  $1s$ ,  $1p$ ,  $1d$ , and  $2s$  while the  $1f$  orbit is occupied by  $(Z - 20)$  protons. According to the assumption of the simple shell model of Eq.(1), an analytical expression for the CDD of  $1f - 2p$  shell nuclei is obtained as [11]:

$$\rho_{ch}(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3} \left[ 5 + 4 \left(\frac{r}{b}\right)^4 + (Z - 20) \frac{8}{105} \left(\frac{r}{b}\right)^6 \right] \dots\dots\dots (2)$$

The calculated results obtained by Eq. (2) have poor agreement with experimental data. To derive an explicit form for the CDD of  $1f-2p$  shell nuclei, we assume that there is a core of filled  $1s$ ,  $1p$  and  $1d$  orbitals and the proton occupation numbers in  $2s$ ,  $1f$  and  $2p$  orbitals are equal to, respectively,  $(2 - \alpha_1)$ ,  $(Z - 20 - \alpha_2)$  and  $(\alpha_1 + \alpha_2)$  and not to 2,  $(Z - 20)$  and 0 as in the simple shell model, where the

parameters  $\alpha_1$  and  $\alpha_2$  are the occupation number of higher shells. Using this assumption, with the help of Eq. (1), the ground state charge density distribution can be written as:

$$\rho_{ch}(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3} \left[ 5 - \frac{3}{2} \alpha_1 + \left(\frac{11}{3} \alpha_1 + \frac{5}{3} \alpha_2\right) \left(\frac{r}{b}\right)^2 + \left(4 - 2\alpha_1 - \frac{4}{3} \alpha_2\right) \left(\frac{r}{b}\right)^4 + \left(\frac{4}{21} \alpha_2 + \frac{8}{105} (Z - 20) + \frac{4}{15} \alpha_1\right) \left(\frac{r}{b}\right)^6 \right] \dots\dots\dots (3)$$

where  $Z$  is the atomic number of nuclei,  $b$  is the harmonic oscillator size parameter.

The normalization condition of the  $\rho_{ch}$  is given by

$$Z = 4\pi \int_0^\infty \rho_{ch}(r) r^2 dr \dots\dots\dots (4)$$

and the mean square radius (MSR) of the nuclei is given by

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho_{ch}(r) r^4 dr \dots\dots\dots (5)$$

The central CDD,  $\rho(r = 0)$  is obtained from Eq. (3) as

$$\rho_{ch}(0) = \frac{1}{\pi^{3/2} b^3} \left[ 5 - \frac{3}{2} \alpha_1 \right] \dots\dots\dots (6)$$

then  $\alpha_1$  is obtained from Eq. (6) as

$$\alpha_1 = \frac{2}{3} \left[ 5 - \rho_{ch}(0) \pi^{3/2} b^3 \right] \dots\dots\dots (7)$$

Substitution of Eq. (3) into Eq. (5) and after simplification gives:

$$\langle r^2 \rangle = \frac{b^2}{Z} \left[ \frac{9Z-60}{2} + \alpha_1 \right] \dots\dots\dots (8)$$

In Eq's (6) and (8), the values of the central density  $\rho_{ch}(0)$  and  $\langle r^2 \rangle$  are taken from the experiments while the parameter  $b$  is chosen in such a way as to reproduce the experimental root mean square radii of nuclei.

The PMD,  $n(k)$ , of the considered nuclei is studied using two distinct methods. In the first, it is determined by the shell model using the single particle harmonic oscillator wave functions in momentum representation and is given by [16]:

$$n(k) = \frac{b^3}{\pi^{3/2}} e^{-b^2 k^2} \left[ 5 + 4 (bk)^4 + (Z - 20) \frac{4}{105} (bk)^6 \right] \dots\dots\dots (9)$$

$k$  is the momentum of the particle.

Where as in the second method, the  $n(k)$  is determined by the Coherent Density Fluctuation Model (CDFM), where the mixed density is given by [9,10]

$$\rho(r, r') = \int_0^\infty |f(x)|^2 \rho_x(r, r') dx \dots\dots\dots (10)$$

where

$$\rho_x(r, r') = 3\rho_0(x) \frac{j_1(k_F(x)|\bar{r} - \bar{r}'|)}{k_F(x)|\bar{r} - \bar{r}'|} \theta(x - \frac{1}{2}|\bar{r} + \bar{r}'|) \dots\dots\dots (11)$$

is the density matrix for  $Z$  protons uniformly distributed in a sphere with radius  $x$  and density  $\rho_0(x) = 3Z / 4\pi x^3$ . The Fermi momentum is defined as [9,10]:

$$k_F(x) = \left( \frac{3\pi^2}{2} \rho_0(x) \right)^{1/3} \equiv \frac{V}{x}; \quad V = \left( \frac{9\pi Z}{8} \right)^{1/3} \dots\dots\dots (12)$$

and the step function  $\theta$ , is defined by

$$\theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases} \dots\dots\dots (13)$$

The diagonal element of Eq. (10) gives the one-particle density a

$$\rho_{ch}(r) = \rho_{ch}(r, r')|_{r=r'} = \int_0^\infty |f(x)|^2 \rho_x(r) dx \dots\dots\dots (14)$$

In Eq. (14),  $\rho_x(r)$  and  $|f(x)|^2$  have the following forms [9,10]:

$$\rho_x(r) = \rho_0(x)\theta(x-r) \dots\dots\dots (15)$$

$$|f(x)|^2 = \frac{-1}{\rho_0(x)} \frac{d\rho_{ch}(r)}{dr} \Big|_{r=x} \dots\dots\dots (16)$$

The weight function of Eq. (16), determined in terms of the CDD satisfies the following normalization condition [9,10]

$$\int_0^\infty |f(x)|^2 dx = 1, \dots\dots\dots (17)$$

and holds for monotonically decreasing density CDD distribution, i.e.  $\frac{d\rho_{ch}(r)}{dr} < 0$ .

On the basis of Eq. (14), the PMD,  $n(k)$ , is expressed as [9,10]:

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k) dx, \dots\dots\dots (18)$$

where

$$n_x(k) = \frac{4}{3}\pi x^3 \theta(k_F(x) - |\vec{k}|), \dots\dots\dots (19)$$

is the Fermi-momentum distribution of the system with density  $\rho_0(x)$ . By means of Eqs. (16), (18) and (19), an explicit form for the PMD is expressed in terms of  $\rho_{ch}(r)$  as

$$n(k) = \left( \frac{4\pi}{3} \right)^2 \frac{4}{Z} \int_0^{V/k} \left[ 6\rho_{ch}(x)x^5 dx - \left( \frac{V}{k} \right)^6 \rho_{ch} \left( \frac{V}{k} \right) \right] \dots\dots\dots (20)$$

with normalization condition

$$Z = \int n_{CDFM}(k) \frac{d^3k}{(2\pi)^3} \dots\dots\dots (21)$$

The elastic monopole form factor  $F(q)$  of the target nucleus is also expressed in the CDFM as [9,10]:

$$F(q) = \frac{1}{Z} \int_0^\infty |f(x)|^2 F(q, x) dx \dots\dots (22)$$

where  $F(q, x)$  is the form factor of uniform charge density distribution given by:

$$F(q, x) = \frac{3A}{(qx)^2} \left[ \frac{\sin(qx)}{(qx)} - \cos(qx) \right] \dots\dots\dots (23)$$

Inclusion of the corrections of the nucleon finite size  $F_{fs}(q)$  and the center of mass corrections  $F_{cm}(q)$  in the calculations requires multiplying the form factor of equation (22) by these corrections. Here,  $F_{fs}(q)$  is considered as free nucleon form factor which is assumed to be the same for protons and neutrons. This correction takes the form [17]:

$$F_{fs}(q) = e^{\left( \frac{-0.43q^2}{4} \right)} \dots\dots\dots (24)$$

The correction  $F_{cm}(q)$  removes the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [17]:

$$F_{cm}(q) = e^{\left( \frac{b^2 q^2}{4A} \right)} \dots\dots\dots (25)$$

It is important to point out that all physical quantities studied above in the framework of

the CDFM such as PMD and  $F(q)$ , are expressed in terms of the weight function  $|f(x)|^2$ . Therefore, it is worthwhile trying to obtain the weight function firstly from the CDDs of two parameter Fermi (2PF) and three parameter Fermi (3PF) models extracted from the analysis of elastic electron-nuclei scattering experiments and secondly from theoretical considerations. The CDD's of 2PF and 3PF, respectively are given by [12]

$$\rho_{ch}(r) = \frac{\rho_0}{1+e^{(r-c)/z}} \dots\dots\dots (26-a)$$

$$\rho_{ch}(r) = \frac{\rho_0(1+wr^2/c^2)}{1+e^{(r-c)/z}} \dots\dots\dots (26-b)$$

Introducing Eqs. (26) into Eq. (16), we obtain the experimental weight function  $|f(x)|_{2PF}^2$  and  $|f(x)|_{3PF}^2$  as

$$|f(x)|_{2PF}^2 = \frac{4\pi x^3 \rho_0}{3A z} \left(1 + e^{\frac{x-c}{z}}\right)^{-2} \exp\left(\frac{x-c}{z}\right) \dots\dots\dots (27-a)$$

$$|f(x)|_{3PF}^2 = \frac{4\pi x^2 \rho_0}{3A} \left[ \frac{\left(1 + \frac{wx^2}{c^2}\right) \left(1 + e^{\frac{x-c}{z}}\right) e^{\frac{x-c}{z}}}{z} - \frac{2wx \left(1 + e^{\frac{x-c}{z}}\right)^{-1}}{c^2} \right] \dots\dots\dots (27-b)$$

Moreover, introducing the derived CDD of Eq. (3) into Eq. (16), we obtain the theoretical weight function  $|f(x)|_{th}^2$  as

$$|f(x)|_{th}^2 = \frac{8\pi x^4}{3Zb^2} \rho_{ch}(x) - \frac{16x^4}{3Z\pi^{1/2}b^5} \left\{ \frac{11}{6} \alpha_1 + \frac{5}{6} \alpha_2 + \left(4 - 2\alpha_1 - \frac{4}{3} \alpha_2\right) \left(\frac{x}{b}\right)^2 + \left(\frac{4}{35} (Z - 20) + \frac{2}{5} \alpha_1 + \frac{2}{7} \alpha_2\right) \left(\frac{x}{b}\right)^4 \right\} e^{-x^2/b^2} \dots\dots\dots (28)$$

**Results and Discussion**

The proton momentum distribution  $n(k)$  and elastic electron scattering form factors for some even 1f-2p shell nuclei are studied by means of the CDFM. The distribution  $n_{CDFM}(k)$  of eq. (20) is calculated by means of the CDD obtained firstly from theoretical consideration as in Eq. (3) and secondly from experiments, such as, 2PF and 3PF [12]. The harmonic oscillator size parameter  $b$  is chosen such that to reproduce the measured root mean square radii (*rms*) of nuclei under study and the parameter  $\alpha_1$  is determined by introducing the chosen value of  $b$  and the experimental

central density  $\rho_{exp}(0)$  into Eq. (7), while the parameter  $\alpha_2$  is assumed as a free parameter to be adjusted to obtain agreement with the experimental *CDD*. It is important to remark that when  $\alpha_1 = \alpha_2 = 0$ , Eq.(3) is reduced to that of the simple shell model prediction.

In Table (1), we present the values of the parameters ( $c$  and  $z$ ) and ( $\omega$ ,  $c$  and  $z$ ) used to extract, respectively, *2PF* and *3PF* CDD'S together with central charge densities  $\rho_{exp}(0)$  for  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei. Table (2) displays all parameters needed for calculating  $\rho_{ch}(r)$  of Eq.(3), such as the harmonic oscillator size parameter  $b$  and the calculated parameters of  $\alpha_1$  and  $\alpha_2$  for considered nuclei. Table(3) demonstrates the calculated occupation numbers for 2s, 1f, and 2p shells. The calculated rms  $\langle r^2 \rangle_{cal}^{1/2}$  and those of experimental data  $\langle r^2 \rangle_{exp}^{1/2}$  [12] are displayed in this table as well for comparison. The comparison shows a remarkable agreement between  $\langle r^2 \rangle_{cal}^{1/2}$  and  $\langle r^2 \rangle_{exp}^{1/2}$  for all considered nuclei.

**Table (1)**  
**Values of various parameters required by the CDD of 2PF and 3PF models together with  $\rho_{exp}(0)$ .**

Nuclei	Parameters of the experimental CDD [12]				$\rho_{exp}(0) \text{ fm}^{-3}$ [12]
	model	w	C (fm)	z (fm)	
<sup>56</sup> Fe	2PF	----	4.111	0.558	0.07562164
<sup>62</sup> Ni	3PF	-0.209	4.442	0.538	0.07999800
<sup>68</sup> Zn	2PF	----	4.353	0.567	0.07434364

**Table (2)**  
**Calculated parameters used in Eq. (3) for the calculations of the CDD.**

Nuclei	Z	b	$\alpha_1$	$\alpha_2$
<sup>56</sup> Fe	26	2.08	0.8055971	0.4899781
<sup>62</sup> Ni	28	2.069	0.7015126	0.7253149
<sup>68</sup> Zn	30	2.12	0.7021746	0.8497643

**Table (3)**  
**Calculated occupation numbers of 2s, 1f, and 2p shells together with the calculated and experimental rms radii.**

Nuclei	Occupation No. of 2s ( $2 - \alpha_1$ )	Occupation No. of 1f ( $Z - 20 - \alpha_2$ )	Occupation No. of 2p ( $\alpha_1 + \alpha_2$ )	$\langle r^2 \rangle_{cal}^{1/2}$ (fm) obtained eq (8)	$\langle r^2 \rangle_{exp}^{1/2}$ (fm) [12]
<sup>56</sup> Fe	1.194403	5.510022	1.295575	3.822	3.800
<sup>62</sup> Ni	1.298487	7.274685	1.426828	3.845	3.822
<sup>68</sup> Zn	1.297825	9.150236	1.551939	3.979	3.979

Fig.(1) shows the dependence of the CDD (in  $\text{fm}^{-3}$ ) on  $r$  (in fm) for <sup>56</sup>Fe, <sup>62</sup>Ni and <sup>68</sup>Zn nuclei. The blue and red curves are the calculated results using Eq. (3) with  $\alpha_1 = \alpha_2 = 0$  and  $\alpha_1 \neq \alpha_2 \neq 0$ , respectively whereas the filled circle symbols correspond to the experimental data [12]. It is obvious that the form of the CDD represented by Eq. (3) behaves as an exponentially decreasing function, as seen by the red and blue curves for all considered nuclei of Fig.(1). This figure shows that the probability of finding a proton near the central region ( $0 \leq r \leq 2$  fm) of the CDD is larger than the tail region ( $r > 2$  fm). Besides, including the higher shells through introducing the values of  $\alpha_1$  and  $\alpha_2$  [presented in Table (2)] into Eq. (3) leads to decreasing significantly the central region of the CDD and increasing slightly the tail region of the CDD, as seen by the red curves. This means that the effect of inclusion of higher shells tends to increase the probability of transferring the protons from the central region of the nucleus towards its surface region and then makes the

nucleus to be less rigid than the case when there is no this effect. Fig.(1) also illustrates that the blue curves in all considered nuclei are not in good agreement with those of experimental data of Ref.[12], especially at the central region of the CDD. But once the higher shells are considered to the calculations, the calculated results for the CDD become in astonishing agreement with those of experimental data throughout the whole range of  $r$  as seen by the red curves.

Fig.(2) illustrates the dependence of the  $n(k)$  (in  $\text{fm}^{-3}$ ) on  $k$  (in  $\text{fm}^{-1}$ ) for <sup>56</sup>Fe, <sup>62</sup>Ni and <sup>68</sup>Zn nuclei. The blue curves correspond to the PMD's of Eq. (9) evaluated by the shell model using the single particle harmonic oscillator wave functions in the momentum space. The filled circle symbols and red curves correspond to the PMD's obtained by the CDFM of Eq. (20) employing the experimental and theoretical CDD, respectively. It is evident that the behavior of the blue curve estimated by the shell model is in contrast with the curves imitated by the

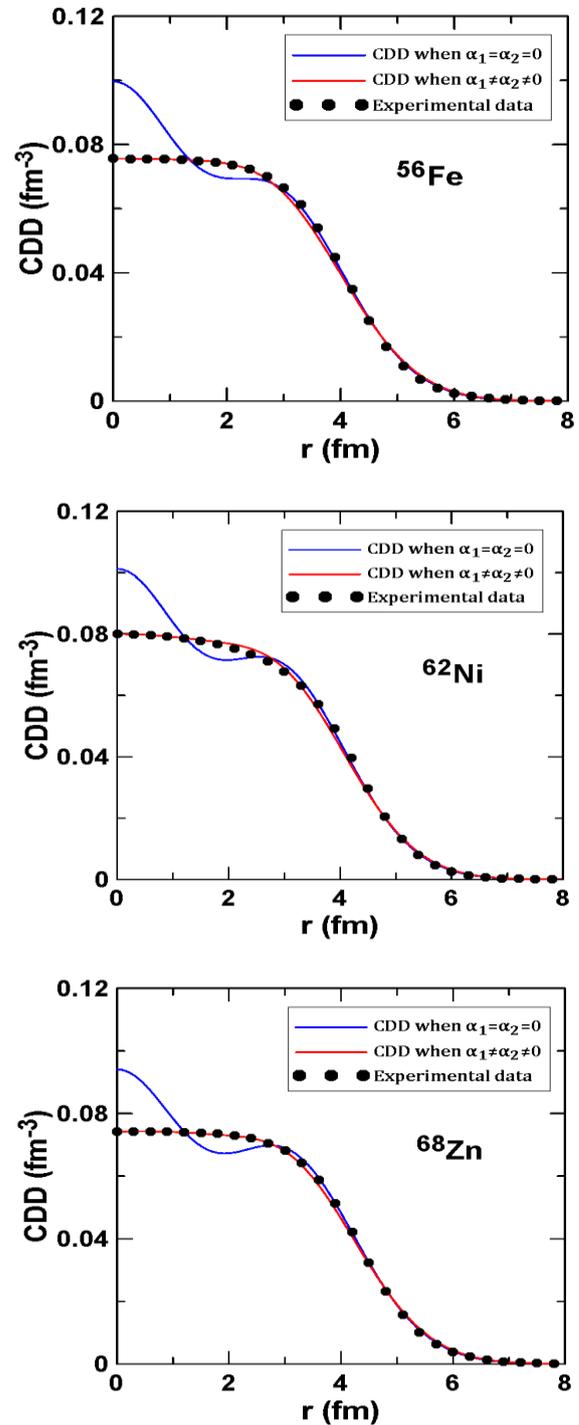
CDFM. The significant property of the blue curve is the steep slope mode, when  $k$  increases. This behavior is in disagreement with the studies [9,10,18-20] and it is attributed to the fact that the ground state shell model wave functions given in terms of a Slater determinant does not take into account the important effects of the short range dynamical correlation functions. Hence, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the PMD [18, 19]. It is noted that the general structure of the filled circle symbols and red curves at the region of high momentum components is almost the same  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei, where these curves have the property of long tail manner at momentum region  $k \geq 2 \text{ fm}^{-1}$ . The property of long-tail manner obtained by the CDFM, which is in agreement with the studies [9, 10, 18- 20], is connected to the presence of high densities  $\rho_x(r)$  in the decomposition of Eq. (14), though their fluctuation functions  $|f(x)|^2$  are small.

The dependence of elastic electron scattering charge form factors on the momentum transfer  $q$  (in  $\text{fm}^{-1}$ ) for considered nuclei is shown in Fig.(3). The calculated form factors (solid curves) of  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei, obtained CDFM using the theoretical weight function of Eq.(28), which are compared with those of experimental data (filled circle symbols) [12,21,22]. This figure shows that the experimental form factors of these nuclei are in very good agreement with those of calculated result.

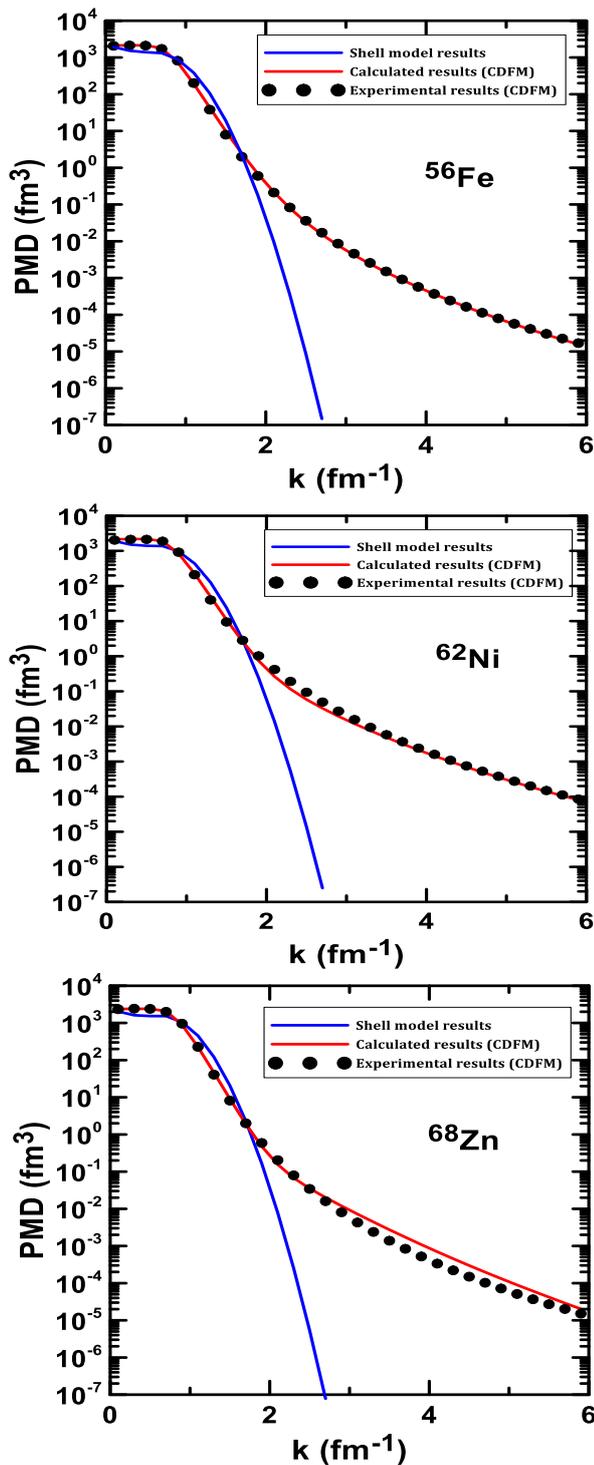
### Summary and Conclusions

The PMD and elastic charge form factors  $F(q)$ , which are evaluated by the CDFM, are formulated via the weight function ( $|f(x)|^2$ ). The weight function, which is related with the local density  $\rho_{ch}(r)$ , is obtained from experiment and from theory. The property of the long-tail mode of the PMD, which is in agreement with the other studies [9,10,18-20], is achieved by both theoretical and experimental weight functions and is connected to the presence of high densities  $\rho_x(r)$  in the decomposition of Eq. (14), though their weight functions are small. It is observed that the theoretical CDD of Eq. (3) utilized in

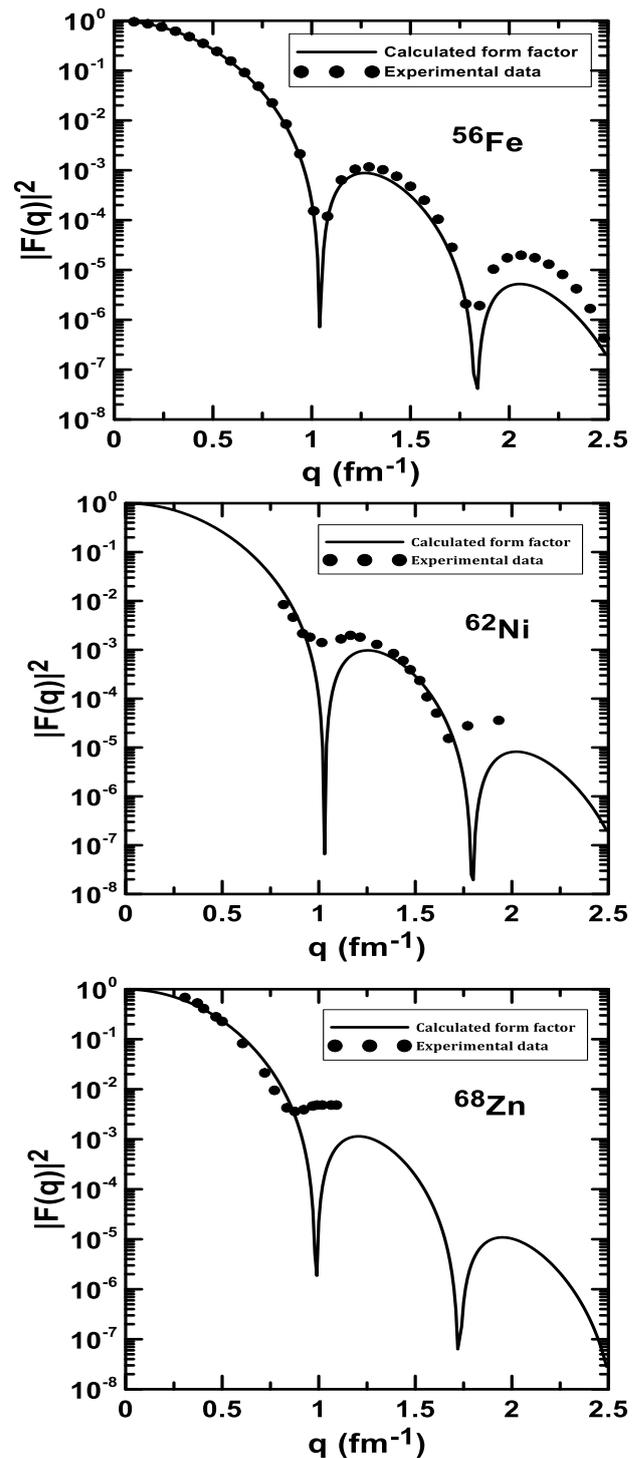
obtaining the theoretical weight function of Eq.(28) is able to provide information about the PMD and elastic charge form factors as do those of the experimental data.



**Fig.(1):** Dependence of the CDD on  $r$  for  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei. The blue and red curves are the calculated CDD of Eq. (3) when  $\alpha_1 = \alpha_2 = 0$  and  $\alpha_1 \neq \alpha_2 \neq 0$ , respectively. The filled circle symbols are the experimental data taken from ref. [12].



**Fig.(2):** Dependence of PMD on  $k$  for  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei. The red curves and filled circle symbols are the calculated PMD obtained in terms of the CDFM of Eq. (20) using the theoretical CDD of Eq. (3) and the experimental data of ref. [12], respectively. The blue curves are the calculated PMD of Eq. (9) obtained by the shell model calculation using the single-particle harmonic oscillator wave functions in momentum representation.



**Fig.(3):** Dependence of the form factors on momentum transfer  $q$  for  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$  nuclei. The solid curve is the calculated form factor of Eq. (22). The experimental data (filled circle symbols) are taken from Refs. [12], [21] and [22] for  $^{56}\text{Fe}$ ,  $^{62}\text{Ni}$  and  $^{68}\text{Zn}$ , respectively.

## References

- [1] Bohr A. and Mottelson B. R., "Nuclear Structure", vol.1, Singapore: World Scientific, 1969.
- [2] Tanihata I., "Neutron halo nuclei", *Journal of Physics G* 22, 157-198, 1996.
- [3] Batty C. J., Friedman E., Gils H. J., and Rebel H., "Experimental methods for studying nuclear density distributions", *Advances in Nuclear Physics* 19, 1-188, 1989.
- [4] Hofstadter R., "Electron Scattering and Nuclear Structure", *Reviews of Modern Physics* 28, 214-254, 1956.
- [5] Donnelly T.W. and Sick I., "Elastic magnetic electron scattering from nuclei" *Reviews of Modern Physics* 56, 461-566, 1984.
- [6] Wapstra A. H., Audi G., and Hoekstra R., "Atomic masses from (mainly) experimental data" *Atomic Data and Nuclear Data Tables* 39, 281-287, 1988.
- [7] Uberall H., "Electron Scattering From Complex Nuclei", Part A, Academic Press, New York, 1970.
- [8] Tel E., Okuducu S., Tamr G., Akti and Bolukdemir M. I., "Calculation of radii and density of  $^{7-19}\text{B}$  isotopes using effective Skyrme force" *Communications in Theoretical Physics* 49, 696, 2008.
- [9] Antonov A. N., Hodgson P. E. and Petkov I. Z., "Nucleon momentum and density distribution in nuclei", Clarendon, Oxford, 1-165, 1988.
- [10] Antonov A. N., Nikolaev V.A., and Petkov I. Z., "Nucleon momentum and density distributions of nuclei", *Z. Physik, A* 297, 257-260, 1980.
- [11] Antonov A. N., Hodgson P. E. and Petkov I. Z., "Nucleon correlation in nuclei" Springer-Verlag, Berlin-Heidelberg-New York, 1993
- [12] Vries H. D., Jager C.W., and Vries C., "Nuclear Charge density distribution parameters from elastic electron scattering" *Atomic data and nuclear data tables* 36 (3), 495-536, 1987.
- [13] Reuter W., Fricke G., Merle K. and Miska H., "Nuclear charge distribution and rms radius of  $^{12}\text{C}$  from absolute elastic electron scattering measurements" *Phys. Rev. C* 26, 806-818, 1982.
- [14] Flaih G. N., "The effect of two-body correlation function on the density distributions and electron scattering form factors of some light nuclei", Ph. D. Thesis, University of Baghdad, 1-134, 2008.
- [15] K. S. Jassim, "Nucleon-nucleon realistic interactions in electron scattering with core-polarization effect" Ph.D. Thesis, University of Baghdad, 1-126, 2007.
- [16] Hassan M. A., "A study of nuclear momentum distributions and elastic electron scattering form factors for light nuclei using coherent density fluctuation model" MSc. thesis, University of Baghdad, 1-80, 2010.
- [17] Brown B. A., Radhi R. and Wildenthal B. H., "Electric quadrupole and hexadecupole nuclear excitations from the perspectives of electron scattering and modern shell-model theory", *Phys. Rep.*, 101 (5), 1983, 313-358.
- [18] Moustakidis C. C. and Massen S. E. "One-body density matrix and momentum distribution in s-p and s-d shell nuclei" *Phys. Rev. C* 62, 34318\_1-34318\_7, 2000.
- [19] Traini M. and Orlandini G. "Nucleon momentum distributions in doubly closed shell nuclei", *Z. Physik, A* 321, 479-484, 1985.
- [20] Ri M. D., Stringari S. and Bohigas O., "Effects of short range correlations on one- and two-body properties of nuclei", *Nuclear Physics A* 376, 81-93, 1982.
- [21] Antonov A. N., Kadrev D. N., Gaidarov M. K., Moya de Guerra E., Sarriguren P., Udias J. M., Lukyanov V. K., Zemlyanaya E. V., and Krumova G. Z. "Charge and matter distributions and form factors of light, medium, and heavy neutron-rich nuclei", *Physical Review C* 72, 1-11, 2005.
- [22] Uberall H. and P. Uginčius, Elastic and inelastic electron scattering from nuclear multipole moments in the first-order born approximation *Phys.Rev*178, 1565-1583, 1969.

### الخلاصة

تم حساب توزيعات زخم البروتون (PMD) وعوامل التشكل للاستطارة الالكترونية المرنة للحالة الارضية لبعض النوى الزوجية ( $^{56}\text{Fe}$ ،  $^{62}\text{Ni}$  و  $^{68}\text{Zn}$ ) الواقعة ضمن القشرة النووية  $1f-2p$  وفقاً لامتداد نموذج تموج الكثافة المترابط الذي يعبر عنه بدلالة دالة التموج  $(|f(x)|^2)$ . لقد تم التعبير عن دالة التموج بدلالة توزيعات كثافة الشحنة وتم حسابها من النتائج النظرية والعملية. تميزت نتائج توزيعات زخم البروتون (المستندة على دالة التموج النظرية والعملية) بخاصية الذيل الطويل عند قيم الزخم العالية. أظهرت هذه الدراسة بان عوامل التشكل النظرية تتفق مع النتائج العملية للنوى ( $^{56}\text{Fe}$ ،  $^{62}\text{Ni}$  و  $^{68}\text{Zn}$ ) ولكل قيم الزخم المنتقل.