Simulation Approach for Estimating the Parameters of the First Type Extreme Value Distribution

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Abstract

In this paper, the two parameters of type I extreme value distribution are estimated for the maximum values by using moment's method and the weighted least square method. The simulation approach is used to compare the obtained results of the applied methods in order to get the best method to estimate the parameters, in which the simulation process starts by generating random samples follows the extreme value distribution. This algorithm is based on three samples of the real parameters and with different sample sizes. The results are tabulated for comparison purpose in connection with the mean square error.

Conclusions

Conclusions concerning the scale parameter θ:

The results of Tables (1), (2) and (3) shows that the moment method is the best in estimating the scale parameter θ for small sample size and its iteration, while the method of weighted least squares method Is the best for medium and large sample size, except the results of Table (2) which shows that the moment method is the best for all sample sizes and their iterations.

Conclusions concerning the location parameter λ:

The results of Tables (1), (2) and (3) shows that the moment method is the best in estimating the location parameter λ for all sample sizes and their iterations. [DOI: 10.22401/JNUS.20.4.15]

Keywords: Extreme value distribution type one for maximum, Gumbul distribution, Estimate parameters.

1. Introduction

The extreme value distribution may be considered as one of the most important distributions, which has so many real life applications. The extreme value refers to the maximum or minimum value.

For the extreme value distribution, three types or families of distributions had been introduced by statisticians [1] in which each family represent a family of several distributions, namely:

- Type I: Gumbel-type distribution.
- Type II: Frećhet-type distribution.
- Type III: Weibull-type distribution.

Type I distributions are of extreme value distributions, which is the most important one and has so many applications in different areas and among of these applications using this distribution in the study and applications of atmosphere studies, such as the effect of atmosphere on sea levels, which are used later in ships designing and to predict winds strength. Also, using this distribution in the theory of water sciences as in the last decades researches proved by using this distribution in the prediction of ozone quantity and other applications are also given for this distribution, such as the predication of high and low rates of financial stocks [8][12].

Type II extreme value distribution has no wide range of applications.

The III distributions may be considered as one of the most important distributions to model failure phenomenon's and used in reliability theory, as well as life testing. From the above three types, it is important to notice that the extreme value distribution represent a derived families from the generalized extreme value distribution, and has so many applications that has been used by Gumbel in the study and measuring of water floods (which is specialized in the atmosphere variations and to describe winds movements and directions, pressure, air humidity, temperature, cloud directions, rains, etc.), landing force and marine engineering, as well as, using this distribution in the study and generalization of certain life data.

Let X be a random variable follows the generalization extreme value distribution, then the probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of X are given respectively by[6]:

$$f(x;k,\theta,\lambda) = \frac{1}{\theta}\left[1-\frac{k}{\theta}(x-\lambda)\right]^{\left(\frac{1}{k}\right)-1} \exp\left[-\left(1-\frac{k}{\theta}(x-\lambda)\right)^{\frac{1}{k}}\right]$$

$$F(x;k,\theta,\lambda) = \exp\left[-\left(1-\frac{k}{\theta}(x-\lambda\right)^{\frac{1}{k}}\right],\ k \neq 0 \quad \text{..... (1)}$$

Where:
x is a random variable
k is the shape parameter
θ is the scale parameter
λ is the location parameter

If k = 0, then the generalized extreme value distribution with three parameters will be the generalized extreme value distribution with two parameters, since by letting:

$$\frac{1}{k} = n \quad \text{.. (2)}$$

and taking the limit as $n \longrightarrow \infty$ after substituting eq.(2) back into eq.(1), will yields to:

$$F = \lim_{n\to\infty}\left\{\exp\left[-\left(1-\frac{1}{n\theta}(x-\lambda)\right)^{n}\right]\right\}$$

$$= \exp\left\{-\lim_{n\to\infty}\left(1-\frac{1}{n}\left(\frac{x-\lambda}{\theta}\right)\right)^{n}\right\} \quad \text{................ (3)}$$

Let:

$$t = \frac{x-\lambda}{\theta} \quad \text{.. (4)}$$

Now, substitute eq.(4) into eq.(3), we get:

$$F = \exp\left\{-\lim_{n\to\infty}\left(1-\frac{1}{n}t\right)^{n}\right\}$$

$$= \exp\left\{-\lim_{n\to\infty}\left[C_0^n 1 - C_1^n\left(\frac{1}{n}t\right)+C_2^n\left(\frac{1}{n}t\right)^2 - C_3^n\left(\frac{1}{n}t\right)^3+\ldots\right]\right\}$$

$$= \exp\left\{-\lim_{n\to\infty}\left[1-n\frac{1}{n}t+\frac{n(n-1)}{n\times n}\frac{t^2}{2!}-\frac{n(n-1)(n-2)}{n\times n\times n}\frac{t^3}{3!}+\ldots\right]\right\}$$

$$= \exp\left\{-\lim_{n\to\infty}\left[1-t+\left(1-\frac{1}{n}\right)\frac{t^2}{2!}-\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\frac{t^3}{3!}+\ldots\right]\right\}$$

$$= \exp\left\{-\left[1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\ldots\right]\right\}$$

$$= \exp[-\exp(-t)] \quad \text{.. (5)}$$

Also, substituting eq.(4) into eq.(5), will yields to:

$$F(x;\theta,\lambda) = \exp\left[-\exp-\left(\frac{x-\lambda}{\theta}\right)\right] \quad \text{................ (6)}$$

Equation (6) represent the distribution function of the type I extreme value distribution (or Gumble-type). In order to find the p.d.f., we derive eq.(6) with respect to x, to get[7]:

$$f(x;\theta,\lambda) = \frac{1}{\theta}\exp-\left(\frac{x-\lambda}{\theta}\right)\exp\left[-\exp-\left(\frac{x-\lambda}{\theta}\right)\right] \quad \text{.... (7)}$$

while the reliability function related to this distribution may be defined as follows[9]:

$$R(x) = 1-\exp\left[-\exp-\left(\frac{x-\lambda}{\theta}\right)\right]$$

2. Some Moment's Properties of Type I Extreme Value Distribution:

The n-th order moment has the general form:

$$E(X^n) = \int_{-\infty}^{\infty} x^n f(x)\,dx \qquad \text{X: random variable}$$

x : the value of a r.r. .... (8)

Therefore, after substituting eq.(7) into eq.(8), we get:

$$E(X^n) = \int_{-\infty}^{\infty} x^n \frac{1}{\theta}\exp-\left(\frac{x-\lambda}{\theta}\right)\exp\left[-\exp-\left(\frac{x-\lambda}{\theta}\right)\right]dx$$

and since $y = \exp-\left(\frac{x-\lambda}{\theta}\right)$, then upon taking the natural logarithm to the both sides will yields to:

$\ln y = -\frac{x-\lambda}{\theta} \Rightarrow -\theta\ln y = x-\lambda \Rightarrow x = \lambda - \theta\ln y$

$$dx = -\frac{\theta}{y}\,dy$$

$x = -\infty \Rightarrow y = \infty,\ x = \infty \Rightarrow y = 0$

$$E(x^n) = \int_{\infty}^{0} (\lambda - \theta \ln y)^n \frac{1}{\theta} y e^{-y} \quad . \quad -\frac{\theta}{y}\, dy$$

$$= \int_0^{\infty} (\lambda - \theta \ln y)^n e^{-y}\, dy$$

with n = 1 we get the first moment, n = 2 we get the second moment, i.e.,

$$n = 1 \Rightarrow E(x) = \int_0^{\infty} (\lambda - \theta \ln y) e^{-y}\, dy$$

$$= \int_0^{\infty} \lambda e^{-y}\, dy - \theta \int_0^{\infty} (\ln y) e^{-y}\, dy$$

$$= \lambda + \theta\gamma \quad \text{..} (9)$$

where:

$$\int_0^{\infty} (\ln y) e^{-y}\, dy = -\gamma$$

γ is Euler's constant, [2]

$\gamma = 0.5772156649\ldots$

$$n = 2 \Rightarrow E(X^2) = \int_0^{\infty} (\lambda - \theta \ln y)^2 e^{-y}\, dy$$

$$E(X^2) = \int_0^{\infty} [\lambda^2 - 2\lambda\theta \ln y + \theta^2 (\ln y)^2] e^{-y}\, dy$$

$$= \lambda^2 \int_0^{\infty} e^{-y}\, dy - 2\lambda\theta \int_0^{\infty} (\ln y) e^{-y}\, dy + \theta^2 \int_0^{\infty} (\ln y)^2 e^{-y}\, dy$$

$$= \lambda^2 + 2\lambda\theta\,\gamma + \theta^2 \left(\gamma^2 + \frac{\pi^2}{6} \right)$$

$$= \lambda^2 + 2\lambda\theta\,\gamma + \theta^2\gamma^2 + \theta^2 \frac{\pi^2}{6}$$

where:

$$\int_0^{\infty} (\ln y) e^{-y}\, dy = -\gamma$$

$$\int_0^{\infty} (\ln y)^2 e^{-y}\, dy = \gamma^2 + \frac{\pi^2}{6} \quad \text{.........................} (10)$$

Equation (10) represent the maximum population second moment taken from the first type extreme value distribution, and since the variance is given by:

$$\mathrm{var}(X) = E(X^2) - [E(X)]^2 \quad \text{..........................} (11)$$

Therefore, substituting eqs.(9), (10) in eq.(11) will give:

$$\mathrm{var}(X) = \lambda^2 + 2\lambda\theta\gamma + \theta^2\gamma^2 + \theta 2 \frac{\pi^2}{6} - [\lambda + \theta\gamma]^2$$

$$= \lambda^2 + 2\lambda\theta\gamma + \theta^2\gamma^2 + \theta^2 \frac{\pi^2}{6} - \lambda^2 - 2\lambda\theta\gamma - \theta^2\gamma^2$$

$$= \theta^2 \frac{\pi^2}{6}$$

3. Estimation Methods

In this section, two methods for estimating the parameters of the first type extreme value distribution will be presented for maximum values. These methods are as follows:

3.1 Method of Moments (MOM):

Johan and Bernaolle (1667-1748) may be considered as the first researchers whom used this method in their work, which is one of the most popular methods used in parameter estimation, which is easy to use. The basic idea of this method is to find population moments, M_j, which are then equated to sample moments, m_j, which will results the number of equations to be equal to the number of unknown parameters. These equations may be solved to find the estimated value of the parameters, [3].

If the population moments are given by:

$$M_j = E(x^j)$$

and the sample moments:

$$m_j = \frac{1}{n} \sum_{i=1}^{n} (x_i)^j$$

Also, as it is known, the sample first moment may be defined as:

$$E(x) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

while the second moment:

$$E(x^2) = \frac{1}{n} \sum_{i=1}^{n} x_i^2$$

Therefore, by eq.(9) the population first moment will take the form:

$$M_1 = \lambda + \theta\gamma$$

and the sample first moment:

$$m_1 = \frac{1}{n}\sum_{i=1}^{n} x_i \Rightarrow m_1 = \bar{x}$$

Hence, equating the population first moment M_1 with sample first moment m_1, yields to:

$$\lambda + \theta\gamma = \bar{x} \Rightarrow \lambda = \bar{x} - \theta\gamma \quad \text{......................... (12)}$$

In a similar manner, from eq.(10), the population second moment is given by:

$$M_2 = \lambda^2 + 2\lambda\theta\gamma + \theta^2\gamma^2 + \theta^2\frac{\pi^2}{6}$$

and the sample second moment:

$$m_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

Equating M_2 with m_2, will give:

$$\lambda^2 + 2\lambda\theta\gamma + \theta^2\gamma^2 + \theta^2\frac{\pi^2}{6} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \quad \text{....... (13)}$$

Therefore, substituting eq.(12) into eq.(13) implies to:

$$(\bar{x}-\theta\gamma)^2 + 2\theta\gamma(\bar{x}-\theta\gamma) + \theta^2\gamma^2 + \theta^2\frac{\pi^2}{6} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$$\bar{x}^2 - 2\bar{x}\theta\gamma + \theta^2\gamma^2 + 2\bar{x}\theta\gamma - 2\theta^2\gamma^2 + \theta^2\gamma^2 + \theta^2\frac{\pi^2}{6} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$$\bar{x}^2 + \theta^2\frac{\pi^2}{6} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$$\theta^2 = \frac{6}{\pi^2}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2\right)$$

Hence:

$$\hat{\theta}_{mom} = \frac{\sqrt{6}}{\pi}\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2} \quad \text{.................... (14)}$$

It is notable that, equation (14) represent the estimation of the scale parameter θ.
Finally, substituting eq.(14) into eq.(12) will give:

$$\hat{\lambda}_{mom} = \bar{x} - \frac{\gamma\sqrt{6}}{\pi}\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2} \quad \text{............ (15)}$$

which is the estimation of the parameter λ, [4].

3.2 Weighted Least Squares Method (WLS):

This method will be used to find the estimation of a and b, which are abbreviated as $\hat{a}$ and $\hat{b}$, respectively [5]. This method is based upon find the inverse of the distribution function, which then compared with simple linear regression model given by:

$$y_i = a + bk_i + e_i \quad \text{.. (16)}$$

where e_i is the observation random error and in order to find the inverse of the distribution function, take eq.(6) which may be simplified to find x, as follows:

$$F(x) = \exp\left[-\exp-\left(\frac{x-\lambda}{\theta}\right)\right]$$

$$-\exp-\left(\frac{x-\lambda}{\theta}\right) = \ln(F(x))$$

$$\frac{x-\lambda}{\theta} = -\ln[-\ln(F(x))]$$

$$\therefore\ x = \lambda - \theta\ln[-\ln(F(x))]$$

$$x_i = \lambda - \theta\ln[-\ln(F(x_i))] \ldots \text{........................... (17)}$$

which is the inverse distribution function.
Comparing eqs.(16) and (17), we have:
$y_i = x_i$, $a = \lambda$, $b =-\theta$
$k_i = \ln[-\ln(F(x_i)]$, $i = 1, 2, \ldots, n$

and from eq.(16):

$$e_i = y_i - a - bk_i \quad \text{.. (18)}$$

Now, multiplying eq.(18) by $\frac{1}{y_i}$ and summing its squares, it will take the form:

$$\sum_{i=1}^{n}\left(\frac{e_i}{y_i}\right)^2 = \sum_{i=1}^{n}\left(\frac{y_i - a - bk_i}{y_i}\right)^2 \quad \ldots\text{................. (19)}$$

Let $S = \sum_{i=1}^{n}\left(\frac{e_i}{y_i}\right)^2$, then eq.(19) will be reduced to:

$$S = \sum_{i=1}^{n}\left(\frac{y_i - a - bk_i}{y_i}\right)^2$$

This method has its basis on making S, which is called the weighted function of least squares, to be minimum, as follows:

$$S = \sum_{i=1}^{n}\left(1 - a\frac{1}{y_i} - b\frac{k_i}{y_i}\right)^2$$

since $y_i = x_i$, then:

$$S = \sum_{i=1}^{n}\left(1 - a\frac{1}{x_i} - b\frac{k_i}{x_i}\right)^2 \quad \ldots \tag{20}$$

then by letting $\frac{1}{x_i} = w_i$ and $\frac{k_i}{x_i} = z_i$, we will get:

$$S = \sum_{i=1}^{n}\left[1 - aw_i - bz_i\right]^2 \quad \ldots \tag{21}$$

Deriving eq.(21) with respect to a and then equating the result to zero, yields to:

$$\frac{\partial S}{\partial a} = \sum_{i=1}^{n} -2w_i[1 - aw_i - bz_i] = 0 \quad \ldots \tag{22}$$

and by multiplying by –1/2, one may get with some simplifications:

$$\sum_{i=1}^{n} w_i[1 - aw_i - bz_i] = 0$$

$$\Rightarrow \sum_{i=1}^{n} w_i - a\sum_{i=1}^{n} w_i^2 - b\sum_{i=1}^{n} w_i z_i = 0$$

$$\sum_{i=1}^{n} w_i - b\sum_{i=1}^{n} w_i z_i = a\sum_{i=1}^{n} w_i^2$$

$$\hat{a} = \frac{\sum_{i=1}^{n} w_i - b\sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i^2} \quad \ldots \tag{23}$$

Similarly, deriving eq.(21) with respect to b and equating the result to zero, will yields to:

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{n} -2z_i[1 - aw_i - bz_i] = 0 \quad \ldots \tag{24}$$

Then multiplying eq.(24) by –1/2:

$$\sum_{i=1}^{n} z_i[1 - aw_i - bz_i] = 0$$

$$\Rightarrow \sum_{i=1}^{n} z_i - a\sum_{i=1}^{n} z_i w_i - b\sum_{i=1}^{n} z_i^2 = 0$$

$$\sum_{i=1}^{n} z_i - b\sum_{i=1}^{n} z_i^2 = a\sum_{i=1}^{n} z_i w_i$$

Hence:

$$\hat{a} = \frac{\sum_{i=1}^{n} z_i - b\sum_{i=1}^{n} z_i^2}{\sum_{i=1}^{n} z_i w_i} \quad \ldots \tag{25}$$

Equating eqs.(23) and (25), we get:

$$\frac{\sum_{i=1}^{n} w_i - b\sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i^2} = \frac{\sum_{i=1}^{n} z_i - b\sum_{i=1}^{n} z_i^2}{\sum_{i=1}^{n} z_i w_i}$$

$$\Rightarrow \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i w_i - b\left(\sum_{i=1}^{n} z_i w_i\right)^2 = \sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i - b\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2$$

$$b\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - b\left(\sum_{i=1}^{n} z_i w_i\right)^2 = \sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i w_i$$

$$b\left[\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} z_i w_i\right)^2\right] = \sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i w_i$$

Hence:

$$\hat{b}_{WLS} = \frac{\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i w_i}{\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} w_i z_i\right)^2} \quad \ldots \tag{26}$$

$$\hat{\theta}_{WLS} = \frac{-\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i w_i}{\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} w_i z_i\right)^2} \quad \ldots \tag{27}$$

which represent the estimation of scale parameter.

Substituting eq.(16) into eq.(27), will give:

$$a = \frac{\sum_{i=1}^{n} w_i - \dfrac{\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i w_i}{\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} w_i z_i\right)^2}\left(\sum_{i=1}^{n} w_i z_i\right)}{\sum_{i=1}^{n} w_i^2}$$

$$= \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \sum_{i=1}^{n} w_i \left(\sum_{i=1}^{n} w_i z_i\right)^2 - \sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i \sum_{i=1}^{n} w_i z_i + \sum_{i=1}^{n} w_i \left(\sum_{i=1}^{n} z_i w_i\right)^2}{\sum_{i=1}^{n} w_i^2 \left[\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} w_i z_i\right)^2\right]}$$

$$= \frac{\sum_{i=1}^{n} w_i^2 \left[\sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i^2 - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} w_i z_i\right]}{\sum_{i=1}^{n} w_i^2 \left[\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} w_i z_i\right)^2\right]}$$

$$\hat{a}_{WLS} = \hat{\lambda}_{WLS} = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i^2 - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i^2 \sum_{i=1}^{n} z_i^2 - \left(\sum_{i=1}^{n} w_i z_i\right)^2} \quad ..... (28)$$

which is the estimation of the location parameter.

4. Simulation [10] [11]

The simulation process has been designed in four basic stages, which are important and necessary to find the estimation of the maximum values of the two parameters of type I extreme value distribution.

Stage I (setting the default values):

This stage is one of the most important stages, in which the later stages of the simulation process will depend on. In this stage, we will set:

(i) Specify default values for the parameters:

The default values of the parameters which consists of three cases, namely:

A1: ($\theta = 1$, $\lambda = 5$), A2:($\theta = 3$, $\lambda =$ ١), A3: ($\theta = 2.5$, $\lambda = 10$)

(ii) Choosing the sample size:

Different sizes of the sample has been selected which are proportional with the effect of the sample size on the accuracy of the results obtained by using the two approaches of this paper, where the small sample size is choosing to be n = 10, medium sample size to be n = 50 and large sample size to be n = 100.

(iii) Choosing the number of iterations:

The number of iterations is selected to be R = 500.

Stage II (the samples generation):

In this stage, the sample random data are generated to follow the first type extreme value distribution for maximum values, and as follows:

The inverse of the distribution function of the extreme value distribution is given by eq.(6), which is:

$$F(x;\theta,\lambda) = \exp\left[-\exp-\left(\frac{x-\lambda}{\theta}\right)\right]$$

Simplifying this formula to get eq.(17), hich is the inverse of the distribution function, to be as follows:

$$x = \lambda - \theta \ln(-\ln(F))$$

and by letting U = F, where U is a continuous random variable defined on the interval (0,1), which then yields to:

$$x = \lambda - \theta \ln(-\ln(U))$$

from which it can be used to generate random sample following the first type extreme value distribution for maximum value.

Stage III (evaluating the parameters):

In this stage, the distribution parameters are estimated by using the estimation method proposed in this paper, which are the moment method given by eqs.(14) and (15) and the weighted least squares method given by eqs.(27) and (28).

Stage IV (comparing the results):

This stage is the last one in the formulation of the simulation model, in which a comparison is made between the extreme value distribution parameters estimation results obtained in stage III by using the norm of the mean square error given by:

$$MSE(\hat{p}) = \frac{1}{R}\sum_{i=1}^{R}(\hat{p}_i - p)^2$$

where:

$\hat{p}$ is the estimation of the parameter p

R is the number of sample iterations

5. Simulation Results

Using the proposed estimation methods, the results presented in table (1) are obtained:

Table (1)
The MSE for estimating the two parameters for the first model A1: ($\theta=1$, $\lambda=5$).

N	Method	θ	Best	λ	Best
10	MOM	0.90163 (0.08905)	MOM	5.06467 (0.11022)	MOM
	WLS	1.12323 (0.10143)		5.31054 (0.21904)	
50	MOM	0.97191 (0.01989)	WLS	5.00357 (0.02381)	MOM
	WLS	1.03464 (0.01520)		5.20418 (0.07038)	
100	MOM	0.98883 (0.01101)	WLS	5.00155 (0.01185)	MOM
	WLS	1.02506 (0.00819)		5.19144 (0.05050)	

Table (2)
The MSE for estimating the two parameters for the second model A2: ($\theta=3$, $\lambda=1$).

N	Method	θ	Best	λ	Best
10	MOM	2.70492 (0.80145)	MOM	1.194 (0.99204)	MOM
	WLS	2.57750 (1.35334)		0.43757 (2.64792)	
50	MOM	2.91573 (0.17902)	MOM	1.01107 (0.21434)	MOM
	WLS	2.59262 (0.55659)		0.76744 (3.33437)	
100	MOM	2.96650 (0.09915)	MOM	1.00465 (0.10668)	MOM
	WLS	2.72391 (0.47621)		0.85628 (3.58671)	

Table (3)

The MSE for estimating the two parameters for the third model A3: ($\theta = 2.5$, $\lambda = 10$).

N	Method	θ	Best	λ	Best
10	MOM	2.25410 (0.55656)	MOM	10.16166 (0.68892)	MOM
	WLS	2.79482 (0.61012)		10.34863 (1.02928)	
50	MOM	2.42977 (0.12432)	WLS	10.00895 (0.14885)	MOM
	WLS	2.58630 (0.09326)		10.01445 (0.22928)	
100	MOM	2.47209 (0.06885)	WLS	10.00387 (0.07408)	MOM
	WLS	2.56292 (0.04980)		9.96834 (0.11291)	

6. Recommendations

From the results of this paper, it is recommended to use the MOM to estimate the distribution parameters of real life models for all sample sizes and their iterations.

References

[1] Samuel Kotz and Saralees Nada Rajah, "Extreme value, distribution theory and Application", (2000).

[2] Christian Walck Particle Physics Group Fysikkum University of Stockholm, "Hand-Book on Statistical Distributions, For Experimentalists", Page (59), (1996).

[3] Ani Shobri and Abdul Aziz Jemain, "LQ-Moment for Statistical Analysis of Extreme Events", Journal Modern Applied Statistical Methods, May, Vol. No.1, 228-258, (2007).

[4] Mann N. R., "Point and interval estimation procedures for the two parameter Weibull and extreme value distribution", Tech., Vol.10, No.2, pp.230-256, (1968).

[5] Shanti S. and Balakrishnan N., "Estimation of the location and scale parameters of the extreme value distribution", Hamilton, Ontario, Canada, West Lafayette in 47907, Department of Statistics, Purdue University, USA, July, (1991).

[6] Johnson, NL., Kotz, s., and Balakrishnan, N, "Continuous univariate distribution". 2nd ed., volume 2. wiley, New York.,(1995).

[7] Handbook of statistical distribution with application, Extreme value distribution page 269 – 273.

[8] Fahad M. Al-Aboud, "Bayesian estimations for the extreme value distribution using progressive censored data and A symmetric loss", Department of Statistics, Faculty Science, King Abdul-Aziz University, (2009).

[9] Lye L. M., Hapvarachechi K. P. and Royane S., "Bayes estimation of the extreme value reliability function", IEEE transactions on Reliability, Vol.42, No.4, pp.641-644, (1993).

[10] Janes, R.T "Simulalion An Modeles Approach" John Wiley and Sons. Inc, (2000).

[11] Diane, I.G. and Lonnie, C. V. "A Simulation Study of Estimators for the 2-parameter Weibull Distribution" IEEE Transaction on Reliability, Vol. R-30, No. 1, (1981).

[12] Engineering statistical handbook 1.3.6.6.16. Extreme value type I distribution.